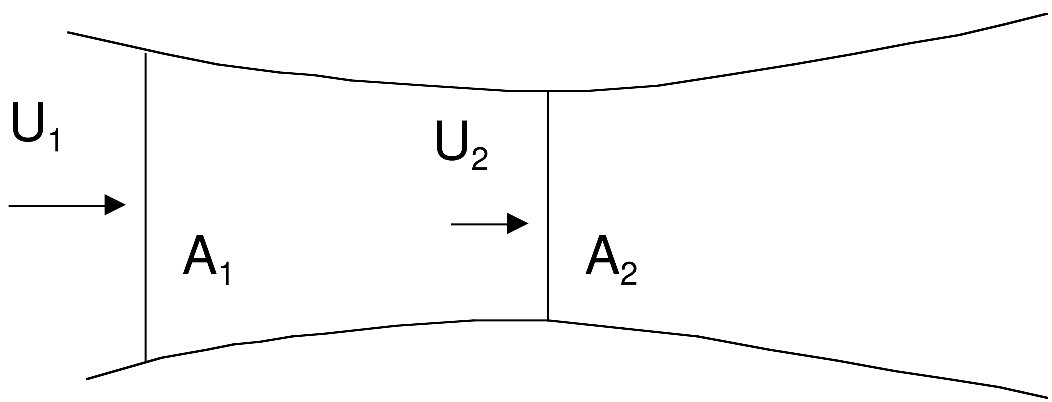
Sound Waves.

To find speed of sound in terms of pressure, p, density, ρ, and temperature T of a gas – need 3 equations of *Fluid Mechanics* – only an outline of these given here.

1. Conservation of Fluid Mass.

Example: → liquid in steady flow through pipe of changing cross-section ⇒ $U_1A_1=U_2A_2$



ie liquid speeds up as pipe narrows.

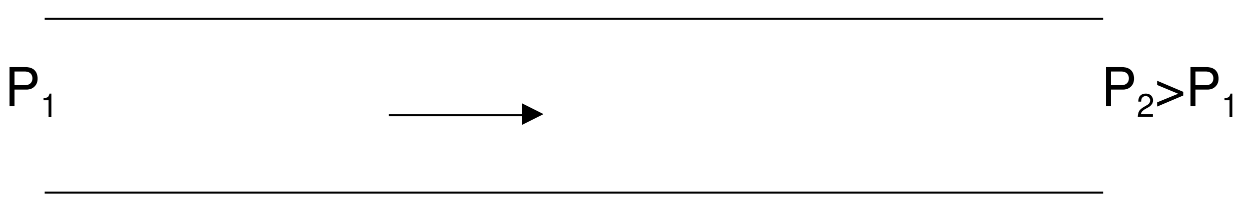
For steady gas flow in pipe ⇒ $\rho_1U_1A_1=\rho_2U_2A_2$

(but full Mass Cons. equation not given here)

2. Newton's Second Law.

Apply to motion of gas particle.

Acceleration of particle $\propto$ - Pressure Gradient



Gas decelerates in pipe if $P_2 > P_1$ but no acceleration or deceleration if $P_2 = P_1$

Gas acceleration also caused by external forces (eg gravity) – ignore for the moment.

3. Gas Equation of State.

Ideal Gas $\Rightarrow$ $p/\rho = RT/m$

where R is Universal Gas Constant and m is gas molecular weight.

If no heat exchange between gas particles allowed $\rightarrow$ adiabatic gas (Sect. 16.8 of Tipler)

$\Rightarrow$ pV^{γ}=constant or $p = k\rho^{\gamma}$

where k is a constant and $\gamma = 5/3$ for Hydrogen gas and 7/5 for air (not proven!)

To obtain speed of sound, c_0, then write

$\rho=\rho_0+\delta\rho = \rho_0(1+\delta\rho/\rho_0) = \rho_0(1+s)$ where $\delta\rho$ is always small compared to ρ_0 and $s=\delta\rho/\rho_0$ is the density contrast.

Then linearise and combine the above 3 equations $\rightarrow$

$$\frac{\partial^2 s}{\partial x^2} - \frac{1}{c_0^{\,2}}\frac{\partial^2 s}{\partial t^2} = 0$$

i.e. Wave equation for density contrast, s, with c_0 the speed of sound.

Can be shown that $c_0^{\,2} = \dfrac{dp}{d\rho}$.

Then since adiabatic gas,

$$p = k\rho^{\gamma} \;\Rightarrow\; \frac{dp}{d\rho} = k\gamma\rho^{\gamma-1} \quad = \quad \frac{\gamma p}{\rho}$$

$$\therefore\; c_0 = \sqrt{\frac{\gamma p}{\rho}} \quad = \quad \sqrt{\gamma\frac{R}{m}T}$$ since ideal gas.

So density (and also pressure, velocity) of air is subject to small sound wave perturbations.

Sound waves are longitudinal waves – gas particles travel in same direction as wave.

General Solution of Wave Equation:-

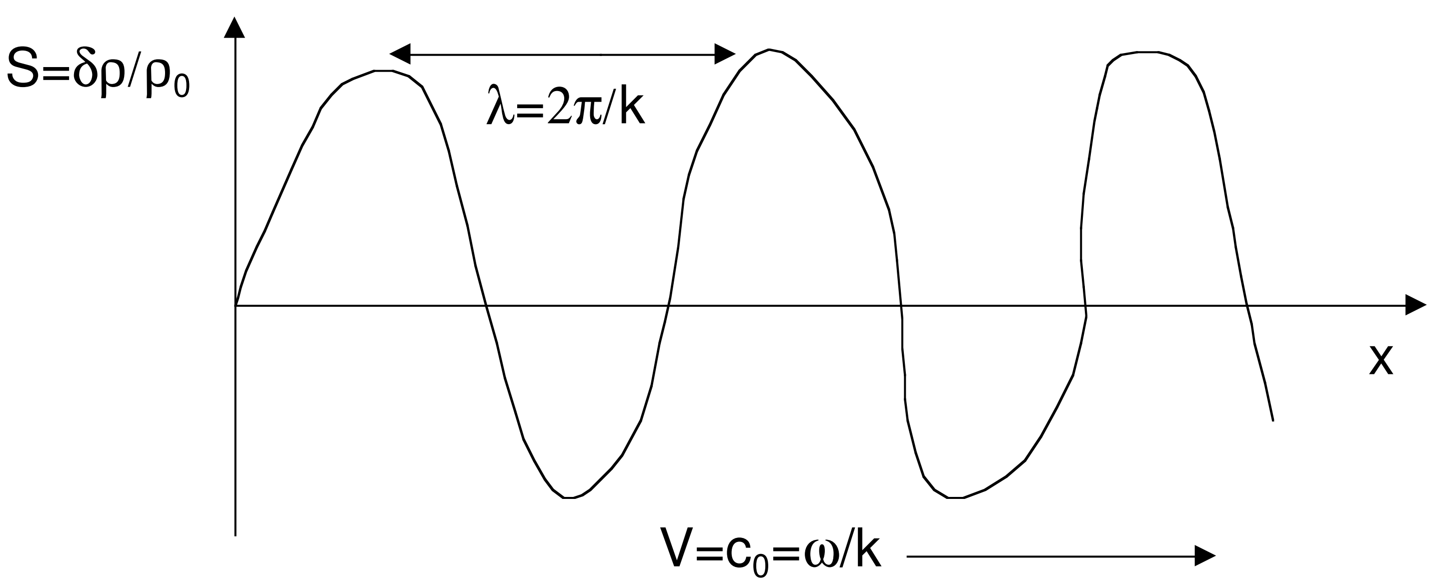
$$\underline{s = F(x - c_0 t) + G(x + c_0 t)}$$

where F and G are arbitrary functions (see Problem Sheet).

Tuning Fork $\Rightarrow$ F is a sine wave

$$\frac{\partial\rho}{\rho_0} = s = A\sin(kx - wt)$$

Exercise: Check this satisfies wave equation with $c_0=\omega/k$



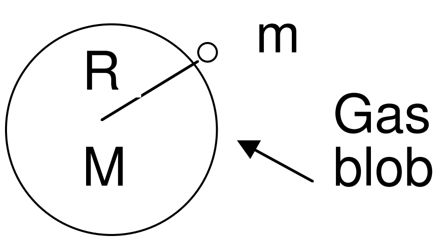
How do sound waves relate to galaxy formation?

When Universe cools to T≈4000K at z=1500, protons and electrons combine and CMB radiation decouples, to leave an expanding mixture of neutral H and He gas.

Gas is prone to sound wave oscillation but oscillation doesn't form galaxies. Extra ingredient needed is gravity.

Concept of Jeans' Scale → when a gas density perturbation is smaller than the Jeans' Scale (R_J) it oscillates like a sound wave. When larger than R_J, the perturbation can grow more dense without oscillating. To calculate R_J need free-fall time.

Free-Fall time of gas particle at edge of spherical gas blob in static Universe:



Newton 2: $\frac{GMm}{r^2} = ma$. Also $s = ut + \frac{1}{2}at^2$ (u=0)

$\therefore \quad R = \frac{1}{2}\frac{GM}{R^2}t^2 \qquad \therefore \; t_{ff} \approx \frac{1}{\sqrt{\rho G}}$ since $\rho \approx \frac{M}{R^3} \approx \rho_0$.

t_{ff} is approximate free-fall or collapse time.

Definition of Jeans' Scale.

Define $R_J \approx c_0 t_{ff}$ where $c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$ and $t_{ff} \approx \frac{1}{\sqrt{G\rho_0}}$.

So if $R<R_J$ then sound wave can cross perturbation within a free-fall time. If $R>R_J$ then sound wave too slow to cross perturbation within t_{ff}, so perturbation can grow denser.

In static Universe, gravitational growth rate is exponential.

In expanding Universe, can show that $10^{11}M_\Theta$ mass galaxies have $R>R_J$ since z=1500, so can grow gravitationally. However, growth rate is slower because gravity has to fight expansion as well as gas pressure.

Result $\rightarrow$ $\underline{\frac{\delta\rho}{\rho} \propto \frac{1}{1+z}}$ in high density, Ω_0=1,

Universe $\rightarrow$ 1500x growth rate since z=1500.

But 10x less grav. growth if Ω_0=0.1

and 100x less grav. growth if Ω_0=0.01.

Can we test if gravity can form galaxies?

Yes $\rightarrow$ using observations of fluctuations in Cosmic Microwave Background.

To form galaxies, need high density contrast by present day i.e. $\frac{\delta\rho}{\rho} \approx 1$.

$\therefore$ at z=1500 when gas was last in contact with microwave background radiation :-

$$\frac{\delta\rho}{\rho}(z=1500) \approx \frac{1}{1500} \quad \text{if } \Omega_0=1$$

$$\frac{\delta\rho}{\rho}(z=1500) \approx \frac{1}{150} \quad \text{if } \Omega_0=0.1$$

$$\frac{\delta\rho}{\rho}(z=1500) \approx \frac{1}{15} \quad \text{if } \Omega_0=0.01.$$

Observations (COBE, Boomerang) of microwave background intensity fluctuations show that $\frac{\delta\rho}{\rho}(z=1500) \leq \frac{1}{3000}$.

$\therefore$ Smoothness of microwave background $\Rightarrow$ we need high density, Ω_0=1, Universe to get $\frac{\delta\rho}{\rho}(z=0) \approx 1$ by present day for gravity to have formed galaxies.