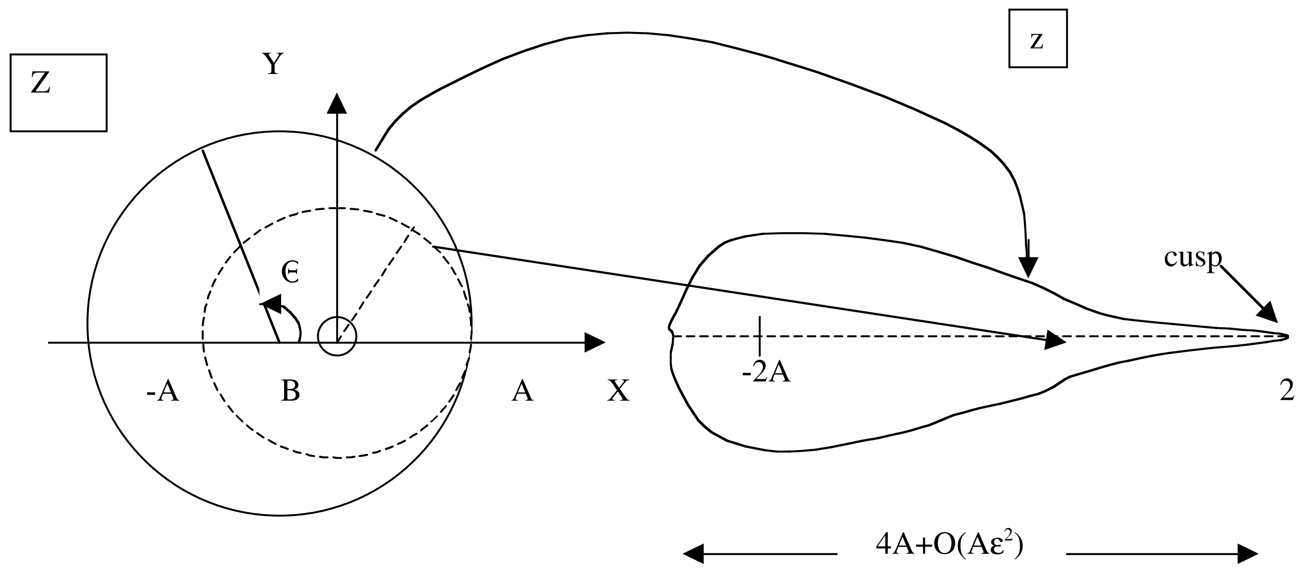
Couple on Elliptical Obstacle. Use Blasius' Formula plus $W_O(Z(z))$ where Z(z) is Joukowski Transform. Obtain $\underline{M = -\frac{1}{2}\pi\rho_0 U^2 \sin 2\alpha\left(a^2 - b^2\right)}$ where a, b are major, minor axes of the ellipse. (not proven).

Map of Eccentric Circles in Z Plane.

1. Centre on OX but not at O, circle passes through A but is outside –A.



Argue from continuity that Z circle just outside critical circle maps close to strip and so has cusp at 2A.

$Z = -B + (A+B)e^{i\Theta}$, Put B=εA, ε<<1.

$$Z = A(1+\varepsilon)e^{i\Theta} - \varepsilon A = Ae^{i\theta}\left[1+\varepsilon\left(1-e^{-i\Theta}\right)\right]$$

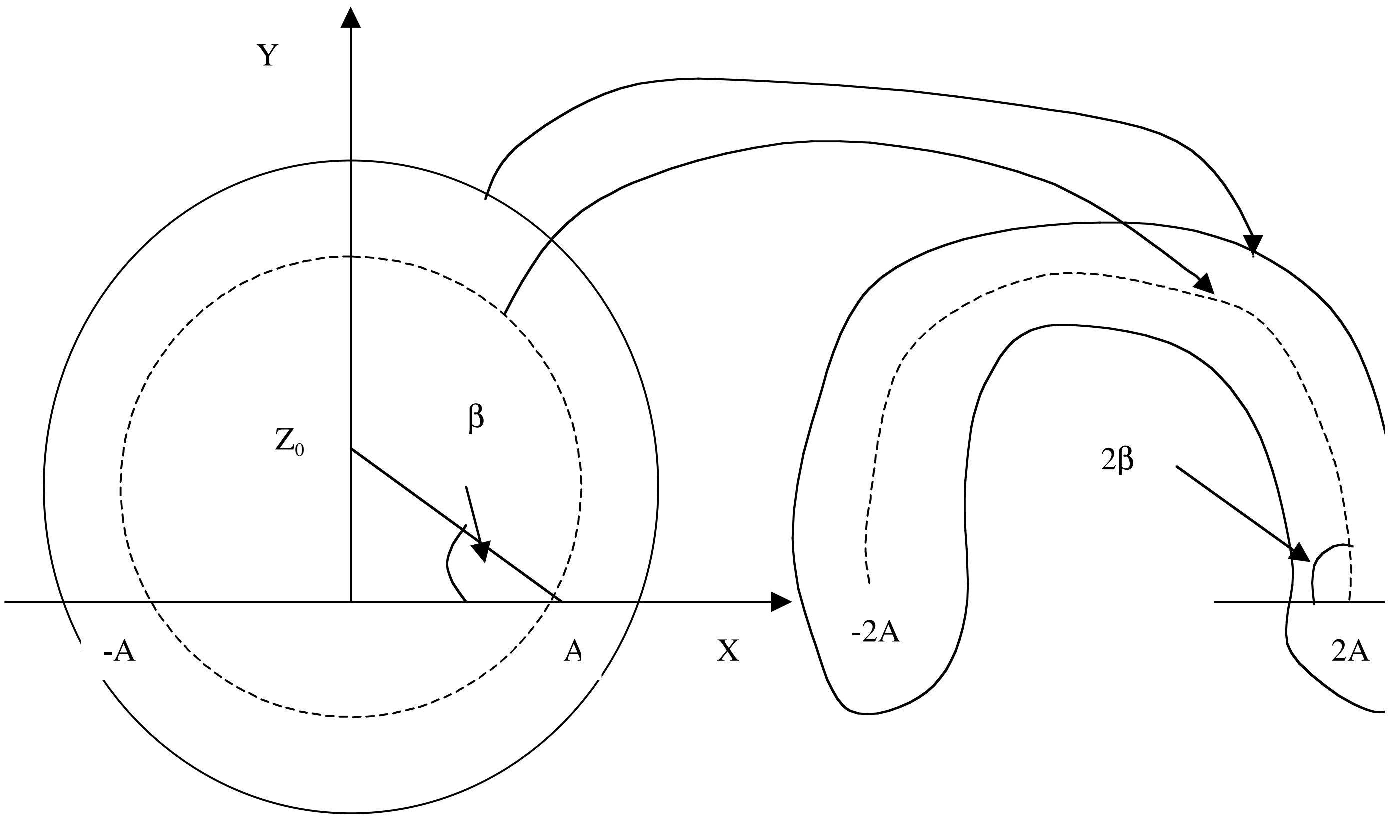
$$\therefore z = Ae^{i\Theta} - \varepsilon A + A\varepsilon e^{i\Theta} + Ae^{-i\Theta}\left[1+\varepsilon\left(1-e^{-i\Theta}\right)\right]^{-1}$$

$$= 2A\cos\Theta + A\varepsilon\left(e^{i\Theta} - e^{-i\Theta} - 1 + e^{-i2\Theta}\right) + ...$$

$$\therefore x = 2A\cos\Theta + A\varepsilon(\cos 2\Theta - 1), \quad y = A\varepsilon(2\sin\Theta - \sin 2\Theta)$$

$\rightarrow$ Parametric form for aerofoil in z plane.

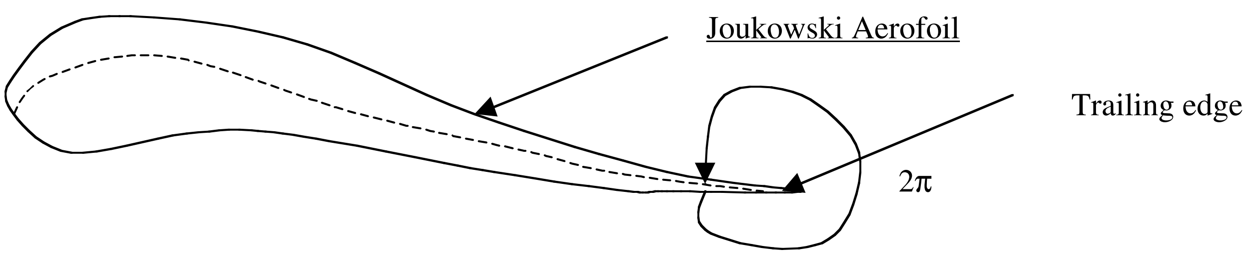
2. Circle centre above O on OY passing through –A, A.



Exterior of dashed circle $\rightarrow$ exterior of circular arc.

Cambered Joukowski Aerofoil.

Combining (1) and (2) above i.e. take circle just outside a slightly off-centre critical circle which passes through A but is just outside –A.



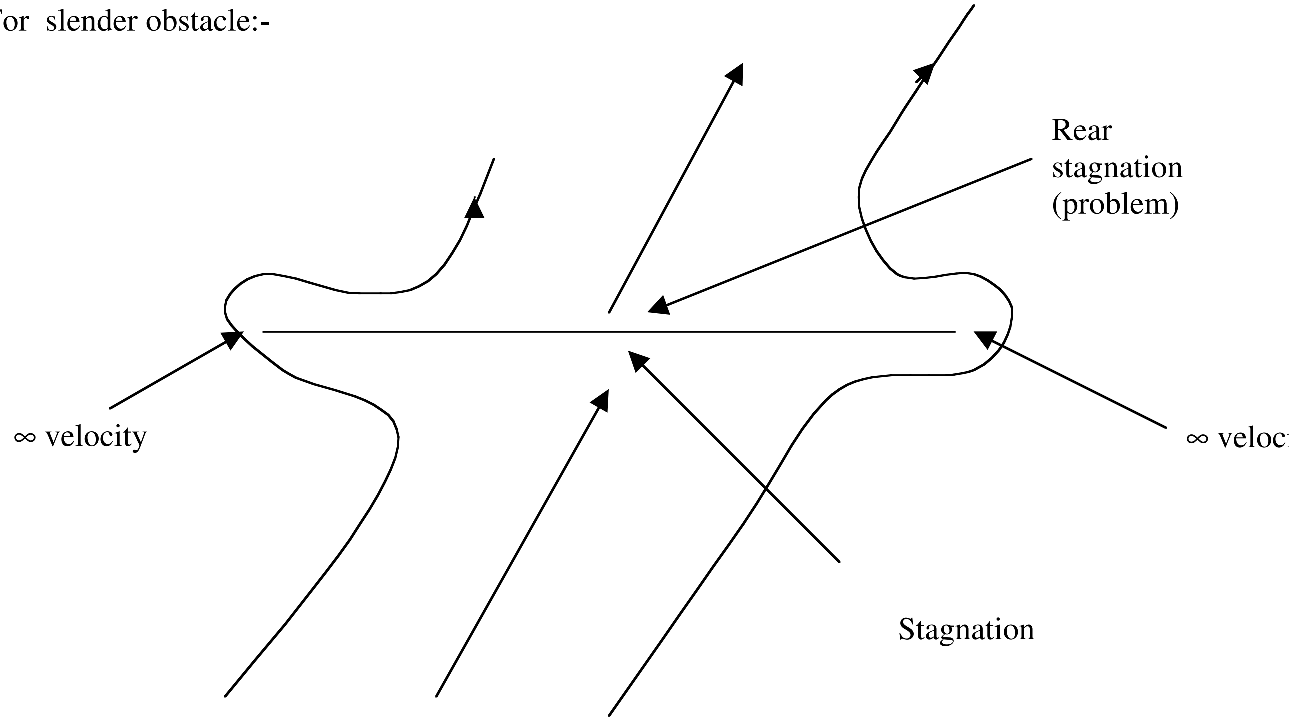
Camber $\equiv$ curved centre line (helps prevent "stalling").

Lift means force perpendicular to flow at ∞ = $-\rho_0 U\Gamma$.
Drag means force parallel to flow at ∞ = 0.

Trailing edge is observed to cause circulation to be set up which causes rear stagnation point to be placed at trailing edge. $\therefore$ Maintains thin boundary layer and stable flow.

Generation of Circulation Around Aerofoil.

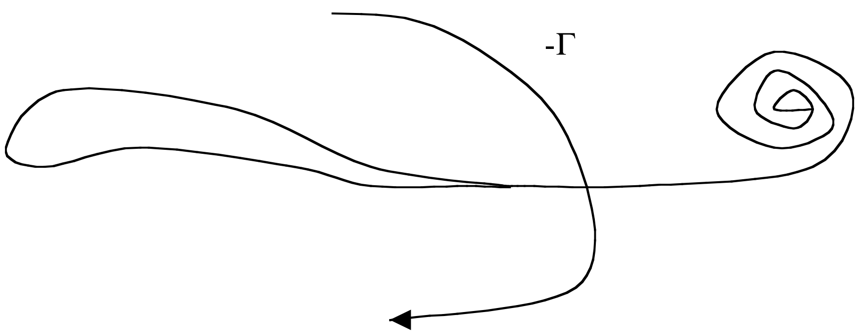
For slender obstacle:-



Rear stagnation and high velocities at edges ($w \sim z^{\frac{1}{2}}$) cause boundary separation $\Rightarrow$ turbulent wake/stalling.

Aerofoil removes front infinity by rounding leading edge.

Induced circulation superpose rear stagnation point and trailing edge infinity to remove both!



Viscous effect at trailing edge causes vortex to be shed at start of motion. Kelvin's Law $\Rightarrow$ circulation set up around aerofoil in opposite direction. Remarkably the amount of circulation is the exact amount to keep flow non-turbulent- viscosity acts to create the circulation which keeps the inviscid solution realistic.

Kutta + Joukowski – determination of circulation which produces stable flow for a given aerofoil shape.

Consider two flows past a wing section with a 2π trailing edge.

(i) $\Gamma \neq 0$ but $U=0 \Rightarrow \quad w \sim cz^{\frac{1}{2}}$ at edge. [wedge]

(ii) $\Gamma = 0$ but $U \neq 0 \Rightarrow \quad w \sim dz^{\frac{1}{2}}$ at edge. [wedge]

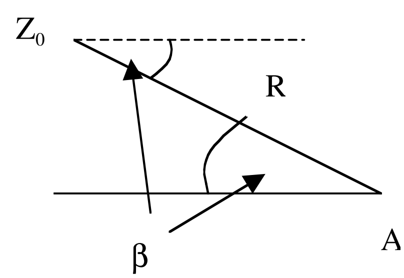
So it is possible to choose Γ such that c+d=0 ie with $\dfrac{dw}{dz}$ finite at trailing edge.

$$\therefore \frac{dw}{dz} = \frac{dW}{dZ}\frac{dZ}{dz} = \frac{\frac{dW}{dZ}}{\frac{dz}{dZ}} = \frac{\frac{dW}{dZ}}{1-\frac{A^2}{Z^2}}.$$

$$\therefore \left.\frac{dw}{dz}\right|_{z=+2A} = \frac{\left.\frac{dW}{dZ}\right|_{Z=A}}{1-\frac{A^2}{A^2}} \quad \text{so we require} \quad \underline{\underline{\left.\frac{\mathrm{dW}}{\mathrm{dZ}}\right|_{Z=A} = 0}}$$

$$\therefore W = Ue^{-i\alpha}(Z-Z_0) + \frac{Ue^{i\alpha}R^2}{Z-Z_0} - \frac{i\Gamma}{2\pi}\ln\frac{Z-Z_0}{R}$$

$$\therefore \frac{dW}{dZ} = Ue^{-i\alpha} - \frac{Ue^{i\alpha}R^2}{(Z-Z_0)^2} - \frac{i\Gamma}{2\pi(Z-Z_0)}$$



Put Z=A and for $\left.\dfrac{dW}{dZ}\right|_A = 0$:- $\quad \dfrac{i\Gamma}{2\pi} = U\left\{(A-Z_0)e^{-i\alpha} - \dfrac{R^2e^{i\alpha}}{A-Z_0}\right\}$

∴ If $A-Z_0=Re^{-i\beta}$, $\quad \therefore \underline{\Gamma = -4\pi UR\sin(\alpha+\beta)} \quad \therefore \quad$ Lift $=-\rho_0 U\Gamma = \underline{4\pi\rho_0 U^2 R\sin(\alpha+\beta)}$

where R and β contain geometry of aerofoil and α is angle of flow inclination.

<u>Notes.</u>

1. For $\alpha \ll 1$, $\beta \ll 1$ and R~A, Lift ~ $\underline{\pi\rho_0 U^2 4A(\alpha+\beta)}$.
2. Lift larger if α or β larger.
3. For (α+β) "large" (ie $\alpha > 10^o$ for $\beta = 8^o$ aerofoil) – aircraft stalls due to high velocity maximum at front → boundary layer separation.
4. Large camber (β) → angle of stall postponed to larger α + larger lift at given α.