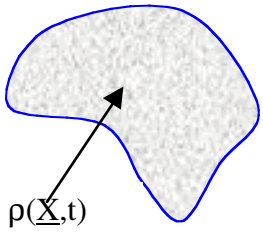
10. Gravitational Instability and Galaxy Formation.

Sir James Jeans (1906) – can stars form by gravitational attraction in an initially homogeneous hydrogen gas with statistical density fluctuations? The same gravitational instability mechanism may also explain galaxy formation in an expanding Universe.

Jeans' Instability. Assume static (non-expanding) Universe field with ideal gas, density ρ, pressure p, velocity $\underline{u}$ and grav. field $\underline{g}$, (body force per unit mass).

Continuity: $\frac{\partial \rho}{\partial t} + \nabla.(\rho \underline{u}) = 0$

Euler: $\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u} = -\frac{1}{\rho}\nabla p + \underline{g}$



Newtonian Gravity: $\nabla.\underline{g} = -4\pi G\rho$ (Poisson's Equation)

(or $\nabla^2 V = 4\pi G\rho$ where $\underline{g} = -\nabla V$).

Unperturbed solution: $\rho = \rho_0, \quad p = p_0, \quad \underline{u} = 0.$

Add small perturbations ρ_1, p_1, $\underline{u}_1$, $\underline{g}_1$ and linearise (similar to soundwaves).

Continuity $\Rightarrow$ $\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla.\underline{u}_1 = 0$ (1)

Euler $\Rightarrow$ $\frac{\partial \underline{u}_1}{\partial t} = -\frac{c_0^2}{\rho_0}\nabla\rho_1 + \underline{g}_1$ (2)

where $c_0^2 = \left.\frac{dp}{d\rho}\right|_0 = \frac{\gamma p_0}{\rho_0} = \frac{5}{3}\frac{R}{m}T$ (monatomic $\Rightarrow$ γ=5/3).

$\nabla.\underline{g} = -4\pi G\rho \quad \Rightarrow \quad \nabla.\underline{g}_1 = -4\pi G\rho_1$ (3)

Eliminate $\underline{u}_1$ in (1) and (2) by taking $\frac{\partial}{\partial t}$ of (1) and $\nabla.$ of (2) obtaining:-

$\frac{\partial^2 \rho_1}{\partial t^2} = c_0^2\nabla^2\rho_1 - \rho_0\nabla.\underline{g}_1 = c_0^2\nabla^2\rho_1 + 4\pi G\rho_0\rho_1$ (from 3).

Solve for Fourier components of ρ_1, $\rho_1 = \sum_{\underline{k}} \delta_{\underline{k}} e^{i\underline{k}.\underline{x}}$.

$\Rightarrow$ for k^{th} Fourier component:-

$$\frac{\partial^2 \delta_{\underline{k}}}{\partial t^2} + \left(k^2 c_0^2 - 4\pi G\rho_0\right)\delta_{\underline{k}} = 0.$$

SHM $\Rightarrow$ $\delta_{\underline{k}} = A_{\underline{k}} e^{-i\omega t}$ (plus $e^{i\omega t}$ $\rightarrow$ decaying solution see below)

where $\omega^2 = \underline{k}^2 c_0^2 - 4\pi G\rho_0$

(<u>dispersion relation</u> - when $\lambda = \frac{2\pi}{k}$ larger $\Rightarrow$ slower wave phase speed, $v_{phase} = \frac{\omega}{k}$).

$$\rho_1 = \sum_{\underline{k}} A_{\underline{k}} e^{i\underline{k}.\underline{x}} e^{-i\omega t} = \sum_{\underline{k}} A_{\underline{k}} e^{i(\underline{k}.\underline{x} - \omega t)}$$

ω is imaginary for wavenumbers, $|\underline{k}|$, below critical value: $k_{Jeans} = \left(\frac{4\pi G\rho_0}{c_0^2}\right)^{\frac{1}{2}}$.

So Fourier components of ρ_1 grow exponentially for $|\underline{k}| < k_{Jeans}$ i.e. for $\lambda > \lambda_{Jeans}$, gravity overcomes gas pressure and density perturbation grows exponentially ($\rho_1 \propto e^{\omega t}$). For $\lambda < \lambda_{Jeans}$, no growth just acoustic oscillation of sound wave.

Growth of Density Perturbations in an Expanding Universe.

Exponential growth rate not applicable in an expanding Universe – gravity has to overcome expansion as well as pressure.

Radiation decouples from matter when temperature of Universe drops to T ≈4000K when Universe was 1/1500 of its present radius, R_0.

($\frac{R_0}{R(t_{dec})} = 1 + z = 1500$ where z is redshift).

Can be shown that perturbations with $\lambda > \lambda_{Jeans}$ grow as $\frac{\rho_1}{\rho} \propto t^{\frac{2}{3}} \left(\propto \frac{1}{1+z} \right)$ in a high density Universe with Ω_0=1 i.e. only power-law rate rather than exponential. Rate is 10x slower in low density Universe (Ω_0=0.1).

Jeans' Scale for H gas at T=4000K.

$c_0 = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma \frac{R}{m} T} \approx 7500 m^{-1} s^{-1}$ (with γ=5/3, m=1).

$\Rightarrow \lambda_{Jeans} = \frac{2\pi c_0}{(4\pi G \rho_0)^{\frac{1}{2}}} = 3 \times 10^{17} m$ given that $\rho_0 \approx 3 \times 10^{-17} kgm^{-3}$ at T=4000K.

$\Rightarrow M_{Jeans} = \frac{4}{3} \pi \lambda_J^3 \rho_0 \approx \underline{10^6 M_O}$ (mass of globular star cluster).

So galaxy sized perturbations (10^{10}-10^{12} M_O) can grow after radiation decouples at z≈1500. To form galaxies by the present day requires $\frac{\rho_1}{\rho} \approx 1$ when collapse to high density occurs. But power-law growth rate only allows at most a 1500x increase in $\frac{\rho_1}{\rho}$ between z=1500 and the present day (z=0), if Ω_0=1. Factor is only 150x for Ω_0=0.1.

∴ Whether gravity can form galaxies depends on amplitude of perturbations at z=1500.

Observations of Microwave Background Anisotropy (e.g. COBE or BOOMERANG).

$\Rightarrow \quad \frac{\delta T}{T} = \frac{1}{3}\frac{\delta\rho}{\rho} \left(= \frac{1}{3}\frac{\rho_1}{\rho} \right)$ at z=1500 (not proven).

Observation $\rightarrow \frac{\delta T}{T} \approx 10^{-4} - 10^{-5}$ at z=1500 on galaxy scale.

$\Rightarrow \quad \frac{\delta\rho}{\rho} \approx 3\times10^{-4} - 3\times10^{-5}$

⇒ full 1500x growth needed for $\frac{\delta\rho}{\rho} \approx 1$ by the present day.

⇒ high density, Ω_0=1 Universe?