Cosmic History

1. The expanding Universe

1.1 Hubble's law

Hubble (1929) measured velocities (using redshifts) and distances (using luminosities) for a sample of galaxies. He found that:

$$v = Hr \tag{1}$$

where v is the recession velocity of the galaxy, r its distance and H the Hubble constant. Experimentally $H = 68 \pm 6$ km/s/Mpc ($1\mathrm{pc} = 3.086 \times 10^{16}\mathrm{m} = 3.26\mathrm{lightyrs}$.

$\rightarrow$ The Universe is expanding

Note. The expansion appears the *same* to all observers at all places in the Universe.

1.2 Measuring distances

For a source of luminosity L,

$$f = \frac{L}{4\pi r^2} \tag{2}$$

where f is the flux (energy/area/time).

Thus, if we:

– know the intrinsic luminosity, L, and
– measure the flux f
$\Rightarrow$ can derive r

By measuring distances and velocities (see below) to galaxies, we estimate H. Once we know H, we can measure distances just using eqn. 2 because measuring velocities us easy, whereas, in general, L is not known.

1.3 The cosmological redshift

Consider a wave train of wavelength λ_e emitted by a light source moving at speed v and observed to have wavelength λ_o. The redshift is defined as:

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e}$$

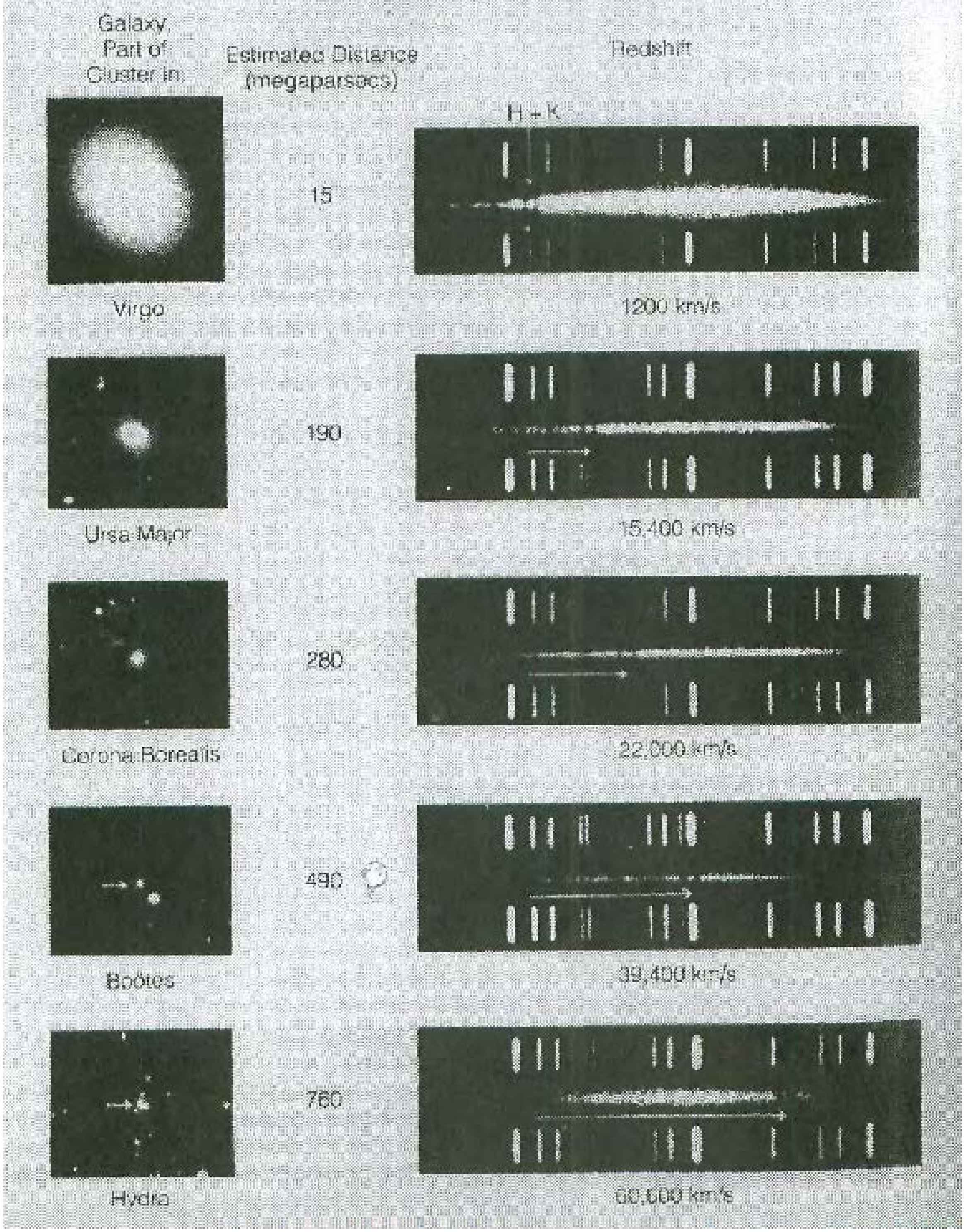
and, for a source moving with $v << c$, is given by:

$$z = \frac{v}{c} \tag{3}$$

The cosmological expansion causes light to be redshifted. Consider a light wave at two epochs, t and t_0 when a patch of universe has size a and a_0 respectively. As the Universe expands, the light wave expands with it, but the number of wave crests remains the same. This is given by:

$$\frac{\lambda}{a} = \frac{\lambda_0}{a_0}$$

Relation Between Redshift and Distance for Remote Galaxies



But,

$$z = \frac{\lambda_0 - \lambda}{\lambda} = \frac{a_0}{a} = 1$$

Thus,

$$\boxed{1 + z = \frac{a_0}{a}} \tag{4}$$

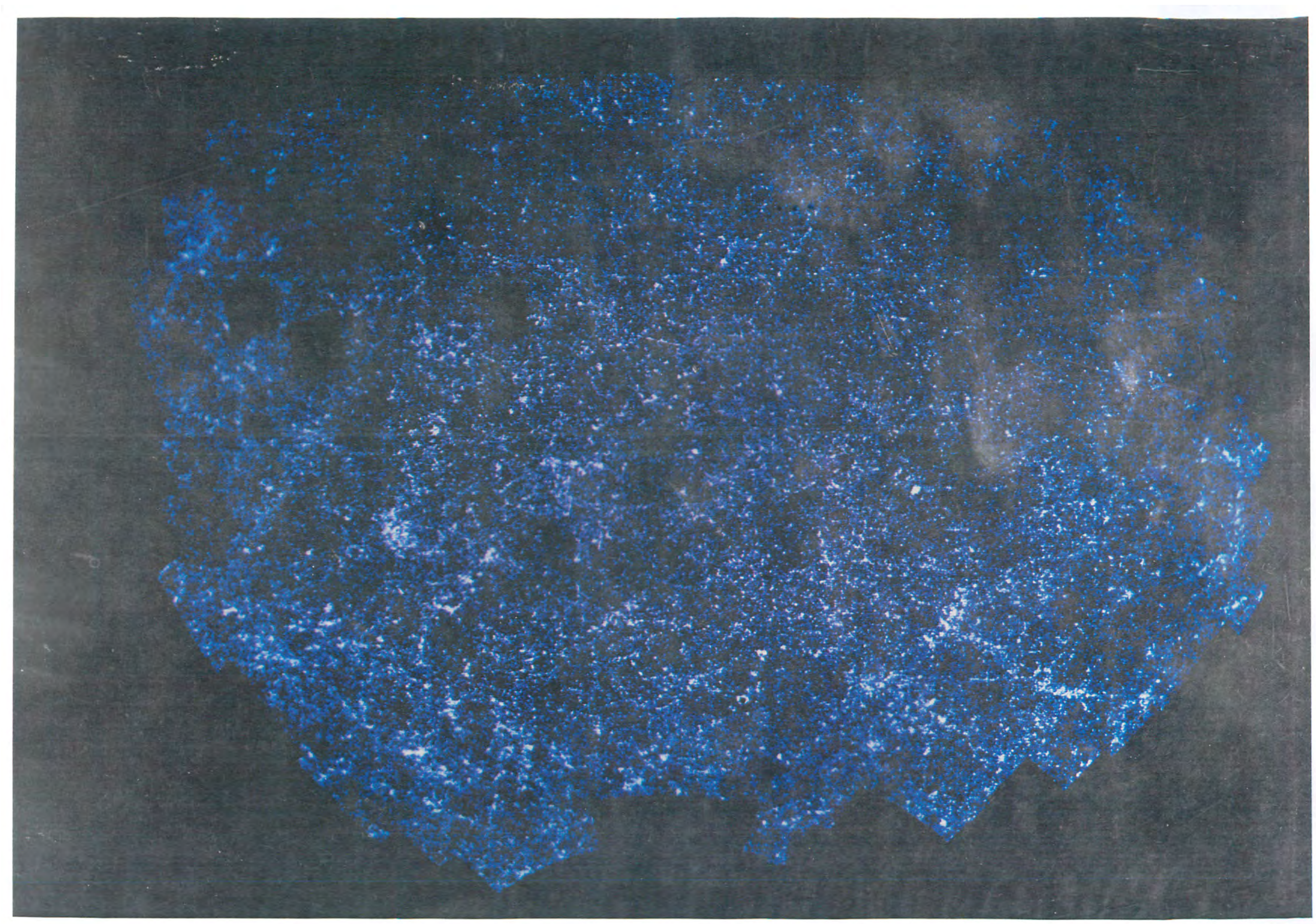
As we look further and further away, we look further and further into the past and thus at hight and higher redshift. Light emitted at the Big Bang would corresponds to $a = 0$ and thus to $z \to \inf$. Today corresponds to $z = 0$.

1.4 The cosmological principle

On large scales ($\gtrsim$100Mpc), the Universe appears *isotropic* (i.e. the same in all directions). For example, galaxy number counts are the same in all directions.

Theorem: If the Universe is locally isotropic about any point, then it is homogenous (i.e. the same from place to place). This can be easily proved graphically (see lectures).

The hypothesis of universal local isotropy (or homogeneity) is known as the *Cosmological Principle*. The hypothesis of homogeneity is also known as the *Copernican Principle*.



1.5 The Friedmann model

That galaxies are flying apart from each other $\Rightarrow$ they were closer together in the past.

Extrapolating to the distant past $\Rightarrow$ an initial state of very large density $\rightarrow$ the Big Bang.

We can derive the equation describing the expansion of the Universe using Newtonian physics. (In reality we would need General Relativity but we can still get the right answer by making some assumptions.)

Consider an arbitrary shell of matter expanding with the Universe. Let r be the radius of the shell, v its expansion velocity and M the mass enclosed within r.

The specific kinetic energy:

$$T \equiv \frac{1}{2}v^2 = \frac{1}{2}H^2 r^2$$

The specific potential energy:

$$W = -\frac{GM}{r}$$

Homogeneity $\Rightarrow$ density is uniform ($\overline{\rho}$) on large scales $\Rightarrow$ $M = \overline{\rho}\frac{4}{3}r^3$ $\Rightarrow W = -\frac{4}{3}\pi G\overline{\rho}r^2$.

$$\text{Energy conservation} \quad \Rightarrow \quad T+W = const \quad \Rightarrow \quad \frac{1}{2}H^2r^2 - \frac{4}{3}\pi G\overline{\rho}r^2 = \text{const} \tag{5}$$

Homogeneity $\Rightarrow$ this equation is valid for *all* shells.

Parametrize r as $r = a(t)r_0$,

where a is known as the dimensionless expansion parameter. Then:

$$\boxed{H^2 = \frac{8\pi}{3}\mathrm{G}\overline{\rho} - ka^{-2}} \tag{6}$$

where k is a constant known as the curvature of the Universe. This is the **Friedmann equation** which describes the expansion of the Universe. General Relativity is required to understand the meaning of k.

Note: $H = \frac{v}{r} = \frac{\dot{a}}{a}$ has units of $1/t$ and is the expansion rate of the Universe.

The Friedmann equation is more difficult to solve than it seems. We can, however, easily derive the general behaviour of the possible solutions.

First rewrite eqn. 6 as:

$$H^2 - \frac{8\pi}{3}\mathrm{G}\overline{\rho} = \frac{8\pi G}{3}\left(\frac{H^2}{8\pi G/3} - \overline{\rho}\right) = -ka^{-2}$$

Define:

$$\overline{\rho}_{crit} = \frac{3H^2}{8\pi\mathrm{G}}, \tag{7}$$

and

$$\Omega = \frac{\overline{\rho}}{\rho_{\text{crit}}} \tag{8}$$

$$\Rightarrow \qquad \frac{8\pi G}{3}(\rho_{\text{crit}} - \overline{\rho}) = \frac{8\pi G}{3}\rho_{\text{crit}}(1-\Omega) = -ka^{-2}$$

There are clearly three possible solutions to this equation:

$$\overline{\rho} > \rho_{\text{crit}} \qquad \text{or} \qquad \Omega > 1 \qquad \Rightarrow \qquad k > 0 \qquad \text{closed} \tag{9}$$

$$\overline{\rho} = \rho_{\text{crit}} \qquad \text{or} \qquad \Omega = 1 \qquad \Rightarrow \qquad k = 0 \qquad \text{flat} \tag{10}$$

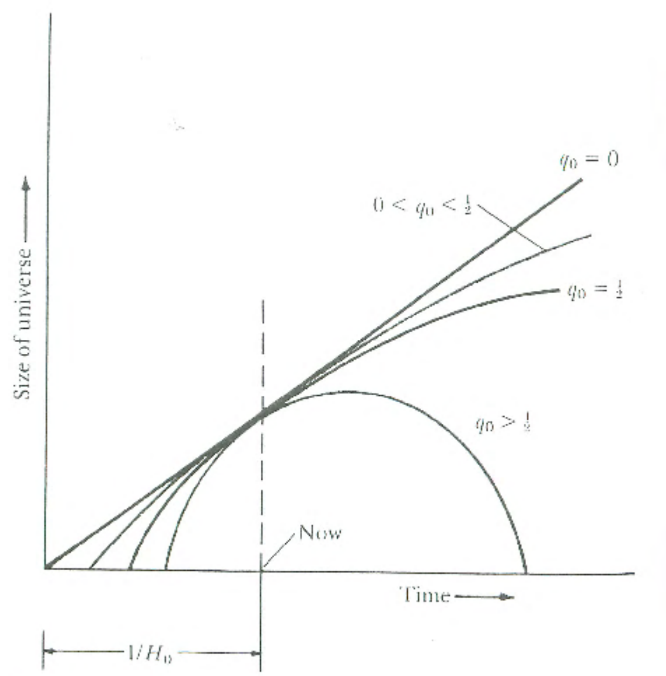
$$\overline{\rho} < \rho_{\text{crit}} \qquad \text{or} \qquad \Omega < 1 \qquad \Rightarrow \qquad k < 0 \qquad \text{open} \tag{11}$$

To see what these solutions look like for a matter-dominated Universe, consider eqn. 6. In a matter-dominated Universe, $\overline{\rho} \propto a^{-3}$. Thus,

• If $k > 0$, at small a, the first term dominates; as a increases, the second term eventually becomes equal to the first and, after that a must decrease because $H^2 > 0$. Thus, the Universe reaches a maximum size and after that it shrinks. Physically, $\overline{\rho} > \rho_{\text{crit}}$ so there is enough gravity to arrest the expansion.

• If $k < 0$, a just increases without bound. Thus, the Universe keeps expanding forever. Physically, $\overline{\rho} < \rho_{\text{crit}}$ so there is not enough gravity to arrest the expansion.

$q = \Omega/2$



• If $k = 0$, from eqn. 5, $T = -W$; the universe coasts to infinity. Physically, $\overline{\rho} = \rho_{\text{crit}}$.

The age of the Universe

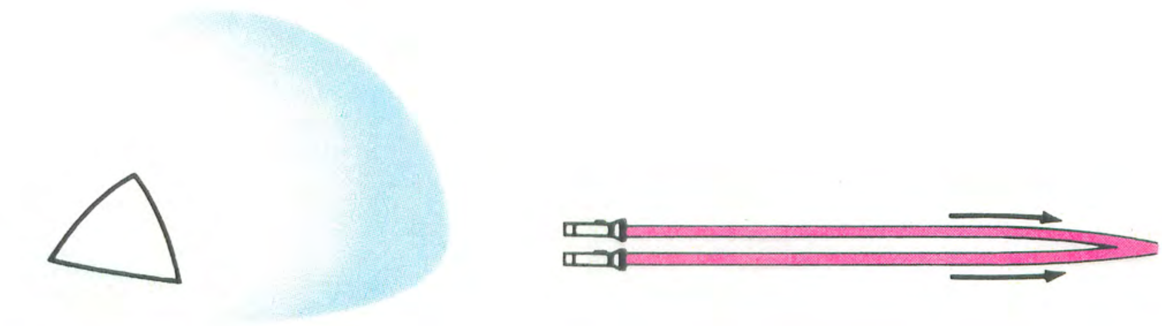
From eqns. 1 and 6, if $\overline{\rho} = 0$, then $v = \text{const}$; the Universe always expands at the same rate, $\dot{a}$, and its age is:

$$\tau = \frac{a}{\dot{a}} = \frac{1}{H} = 1.5 \times 10^{10} \text{ yrs}$$

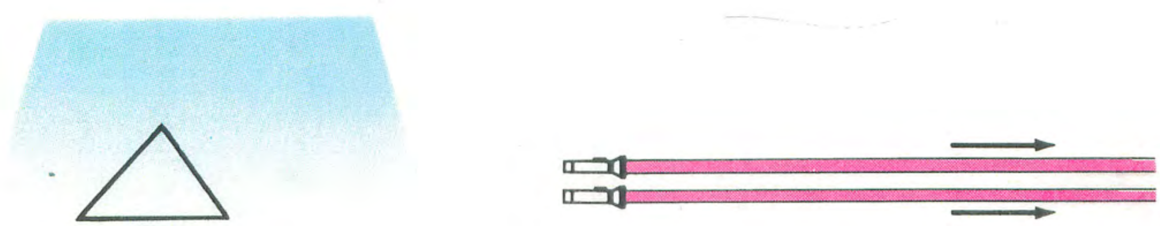
.

In a matter-dominated Universe, $t_0 < t$ because matter decelerates the expansion. For example, for $\Omega_{\text{matter}} = 1$, $\tau = \frac{2}{3}\frac{1}{H_0}$ (see problem sheet).

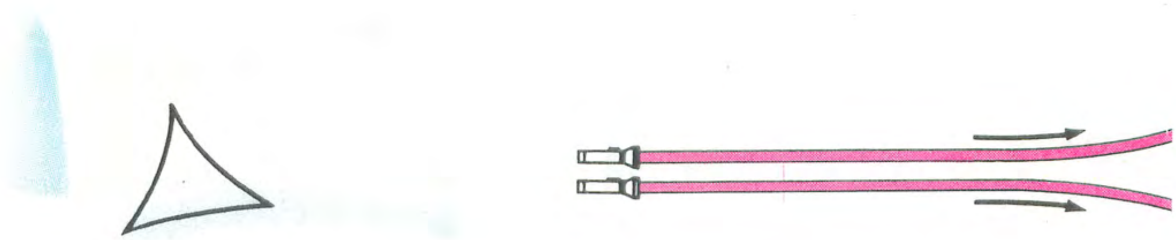
In a dark energy-dominated Universe, the expansion accelerates and the age (at a fixed H_0) is longer than in the matter-dominated case.



a Spherical space $\rho > \rho_c$



b Flat space $\rho = \rho_c$



c Hyperbolic space $\rho < \rho_c$

Fig. 28-13
Kaufmann ***Universe,*** Fourth Edition

1.6 The cosmological constant or dark energy

In 1999 astronomers were able to measure the luminosity of supernovae (SN) of type Ia. These are important because (i) they are bright and (ii) they are "standard candles," that is, they have the same intrinsic luminosity. These SN can thus be used to study the propagation of light over very large distances and, from this, the geometry of space-time.

The SN studied in the late 1990s turned out to be *fainter* than would be expected even if the Universe were empty. This means that their light must have travelled farther than expected. This can only happen if the expansion of the Universe has been accelerating!

The standard Friedmann equation does not admite an accelerating solution. However, if we introduce a new term and make the transformation:

$$\overline{\rho} \rightarrow \overline{\rho} + \frac{\Lambda c^2}{8\pi G} \tag{12}$$

then, for $\Lambda > 0$, we can have an accelerating solution. Introducing the new term is allowed because this term represents a constant energy density and this produces no net force. Λ is the cosmological constant introduced by Einstein (for the wrong reasons) and is sometimes called "dark energy."

Exercise: Show that the Friedmann equation 6 does not admit accelerated expansion but that such an expansion is possible with the transformation in eqn. 12.

<u>Soln.</u>

Friedmann + (L [illegible] ⇒)

$$\dot{a}^2 = \frac{8\pi}{3} G \left(\bar{\rho} + \frac{\Lambda c^2}{8\pi G} \right) a^2 - k$$

$$\dot{a}^2 = \frac{8\pi}{3} G \rho_0 \frac{a_0^3}{a} + \frac{\Lambda c^2}{3} a^2 - k$$

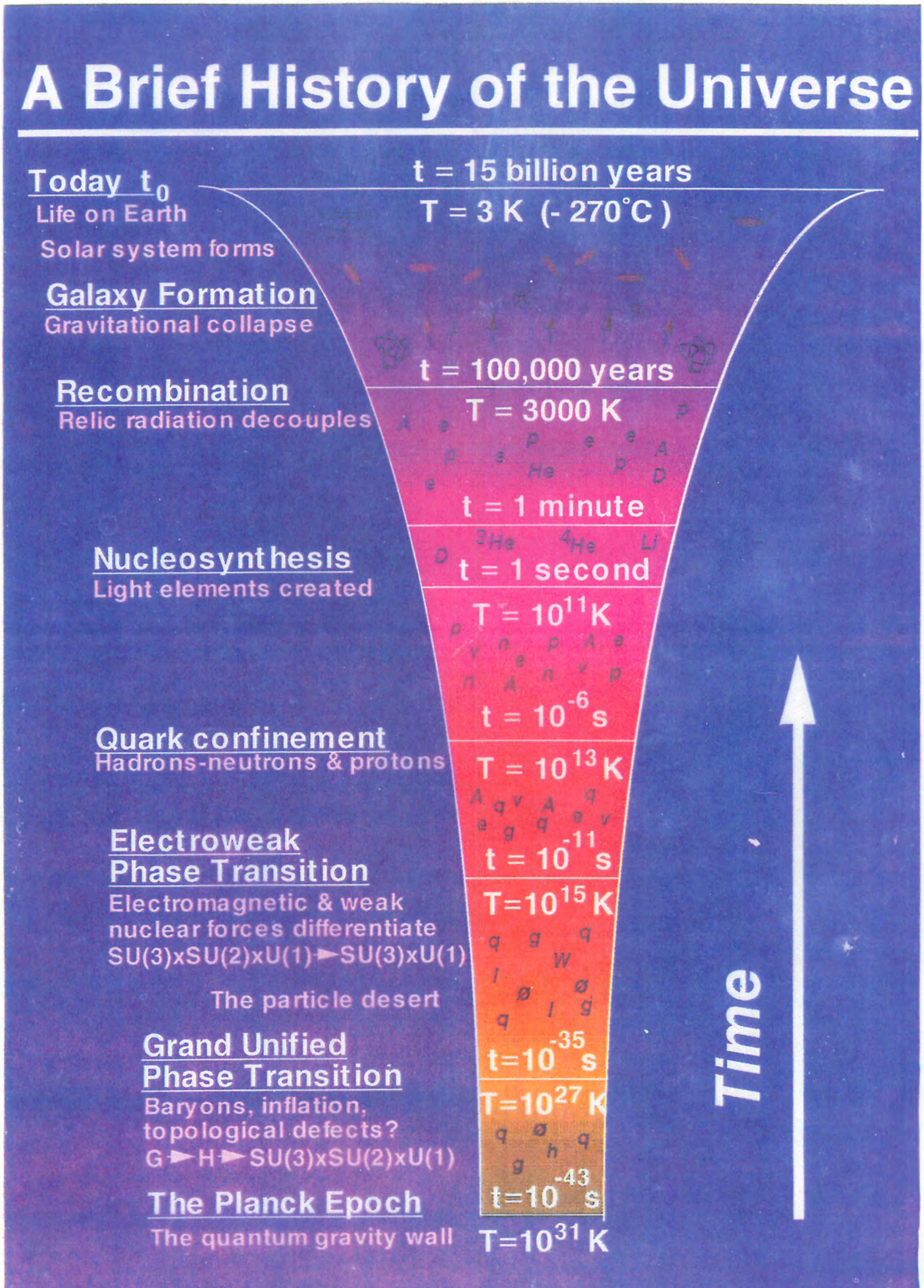
Differentiate w.r.t. t

$$2\dot{a}\ddot{a} = \frac{8\pi}{3} G \rho_0 a_0^3 \left(-\frac{1}{a^2} \right) \dot{a} + \frac{2\Lambda c^2}{3} a\dot{a}$$

$$\Rightarrow \ddot{a} = -\frac{4\pi}{3} G\rho_0 \left(\frac{a_0}{a} \right)^3 a + \frac{\Lambda c^2}{3} a = \frac{4\pi}{3} G \left(\frac{\Lambda c^2}{4\pi G} - \rho \right) a$$

If $\Lambda = 0$, $\quad \ddot{a} = -\frac{4\pi}{3} G \rho a < 0 \quad \Rightarrow$ U decelerates

If $\Lambda \neq 0$ and $\quad \frac{\Lambda c^2}{4\pi G} > \rho$, then $\ddot{a} > 0$ and U accelerates.


A Brief History of the Universe
t = 15 billion years
T = 3 K (- 270˚C)
Today t_0
Life on Earth
Solar system forms
Galaxy Formation
Gravitational collapse
t = 100,000 years
Recombination
Relic radiation decouples
T = 3000 K
t = 1 minute
Nucleosynthesis
Light elements created
t = 1 second
$T = 10^{11}$ K
$t = 10^{-6}$ s
Quark confinement
Hadrons-neutrons & protons
$T = 10^{13}$ K
Electroweak
Phase Transition
Electromagnetic & weak
nuclear forces differentiate
SU(3)xSU(2)xU(1) ► SU(3)xU(1)
$t = 10^{-11}$ s
$T = 10^{15}$ K
The particle desert
Grand Unified
Phase Transition
Baryons, inflation,
topological defects?
G ► H ► SU(3)xSU(2)xU(1)
$t = 10^{-35}$ s
$T = 10^{27}$ K
The Planck Epoch
The quantum gravity wall
$t = 10^{-43}$ s
$T = 10^{31}$ K
Time

2. The Big Bang

2.1 Preliminaries

• Particle/antiparticle pairs can be created out of energy (photons) provided

$$E_\gamma > 2m_pc^2 \tag{13}$$

where m_p is the mass of the particle. (m_pc^2 is the "rest mass-energy")

Example: An electron-positron pair be created out of a photon with $E > 2m_pc^2 = 1.02$ MeV.

• When particles and anti-particles come together, they annihilate.

Example: $e^+ + e^- \to \gamma + \gamma$ (with the two photons moving in opposite directions to conserve energy).

• By the "temperature of the Universe" we mean the temperature of the radiation and any particles in equilibrium with the radiation. It can be measured in Kelvinn(K) or as an energy, $E = kT$, where k is the Boltzman constant.

2.2 The thermal history of the Universe

Initially all four forces of Nature are unified (in a single force). As the Universe expands, symmetries are broken and the forces separate out, one at a time.

<u>*The GUT era*</u> ($kT \sim 10^{19} - 10^{15}$GeV; $t \sim 10^{-35}$s)

The strong, weak and electromagnetic forces are unified. Some GUT theories predict non-equilibrium reactions that do not conserve baryon number. This could give rise to a small excess of quarks over anti-quarks which would explain why the Universe is made of matter, rather than having equal amounts of matter and anti-matter.

Observationally, we know that the ratio of the number density of baryons over photons is $n_b/n_\gamma \sim 10^{-9}$ (i.e. for every 10^9 anti-baryons, there are $10^9 + 1$ baryons).

It is not yet established if this sort of mechanism is responsible for the baryon asymmetry of our Universe.

<u>*The Hadron era*</u> ($kT \sim 10^{15} - 1$GeV; $t \sim 10^{-5}$s)

While $kT \geq 1$ GeV, reactions such :

$$\gamma + \gamma \to p^+ + p^1 \tag{14}$$

$$\gamma + \gamma \to n + \overline{n} \tag{15}$$

maintained hadrons in equilibrium. When kT dropped below 1 Gev, however, these reaction no longer occur $\to$ hadrons annihilate, except for the small baryon excess if this was indeed produced during the GUT era. The radiation produced by this (and subsequent) annihilations contribute to today's cosmic microwave background.

The Lepton era ($kT \sim 1$MeV; $t \sim 10^5$s)

During this time the Universe is dominated by photons and leptons. When $kT \sim 1$ MeV, the electrons and positrons annihilated, producing radiation and leaving only a small excess of electrons to ensure charge neutrality.

There are some models in which the baryon asymmetry is produced around this time.

2.3 Big Bang nucleosynthesis (BBNS) ($kT \sim 1\text{MeV}$; $t \sim 10 - 200\text{s}$)

During the lepton era, neutrons are continually being created and destroyed by reactions such as:

$$p + e^- \leftrightharpoons n + \nu$$
$$p + \overline{\nu} \leftrightharpoons e^+ + n$$

(where $\leftrightharpoons$ means that the reactions occur in both directions.

When the e^- and e^+ pairs annihilate (at $\sim 1\text{MeV} \simeq 10^{16}\text{K}$, these reactions stop and neutrons are no longer created or destroyed. As a result the number of neutrons gets *frozen* relative to the number of protons. In thermal equilibrium, the ratio is given by the "Boltzmann factor":

$$\frac{n}{p} = e^{-\frac{(m_n - m_p)c^2}{\text{KT}}} \tag{16}$$

The mass difference between the neutron and the proton is: $m_n - m_p = 1.3\text{MeV}/c^2$. Since KT $\simeq$ 1Mev (0.8MeV to be precise),

$$\frac{n}{p} = e^{-\frac{1.3}{0.85}} \simeq \frac{1}{5} \tag{17}$$

i.e. there is one neutron for every five protons.

Neutrons are unstable so, if left alone, they would decay after about 10 mins. However, much before they all decay, then neutrons react with protons (a nuclear fusion reaction) to produce deuterium:

$$n + p \leftrightharpoons D + \gamma$$

To begin with, the temperature is too high for D to survive and it is photo-dissociated. This phase is called the *deuterium bottleneck*. However, when the Universe has cooled down to $T \simeq 10^9\text{K}$ ($t \sim 100\text{s}$), D is no longer destroyed and we get chain reactions:

$$D + D \longrightarrow \begin{cases} {}^3He + n & ({}^3He = 2p + n) \\ {}^3H + p & ({}^3H = 1p + 2n \text{ known as tritium}) \end{cases}$$

and then,

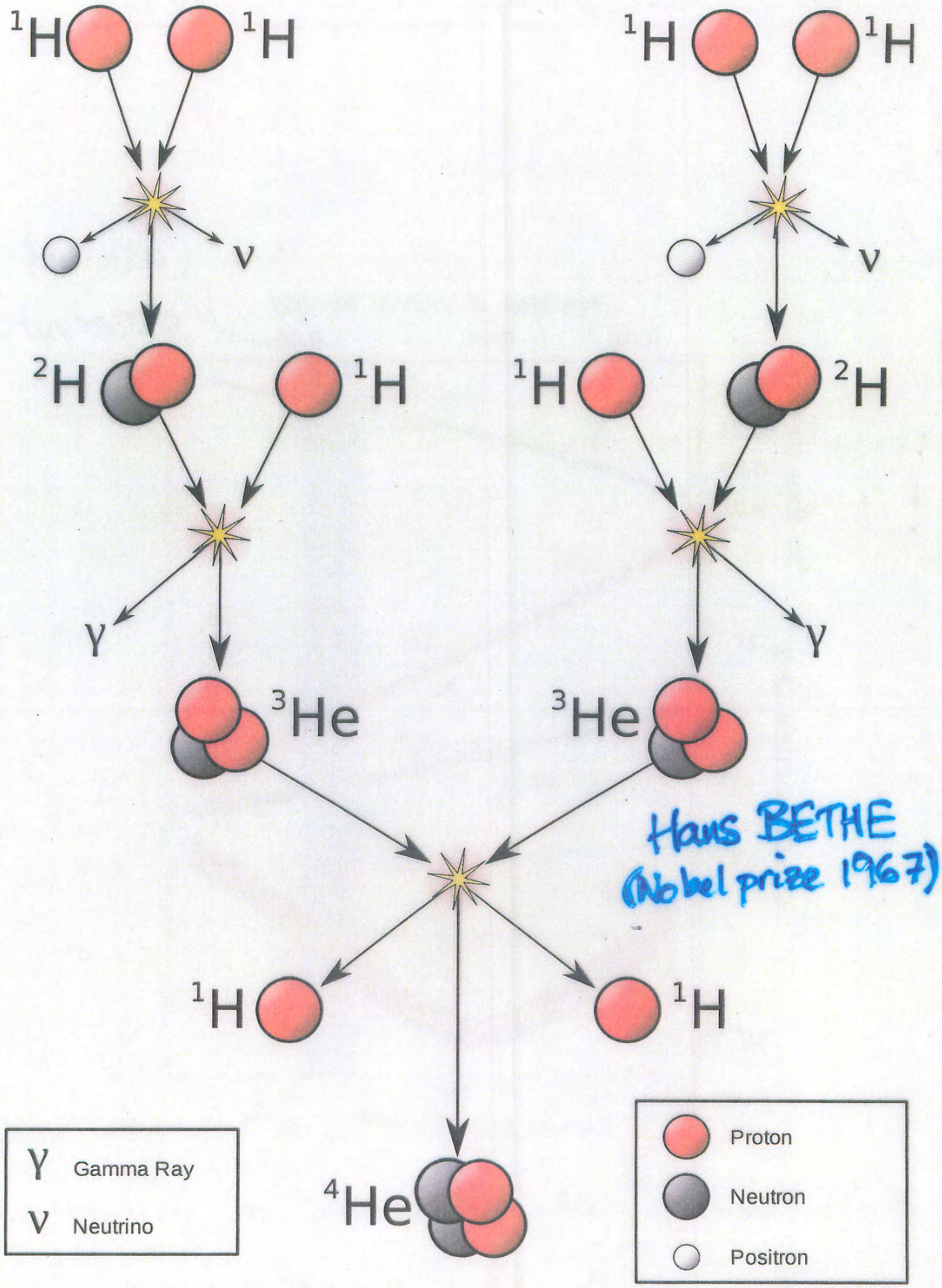
$${}^3He + D \to^4 He + n$$

Practically all the neutrons go into ${}^4He$. A ${}^4He$ nucleus has two protons and two neturons. Therefore the mass fraction of ${}^4He$ is:

$$Y = \frac{\text{mass of} He^4}{\text{total mass}} = \frac{4 \times \text{N}_{\text{He}}}{n + p} = \frac{2n/p}{1 + n/p} \simeq 0.3 \tag{18}$$

A more precise calculation gives $Y = 0.24$

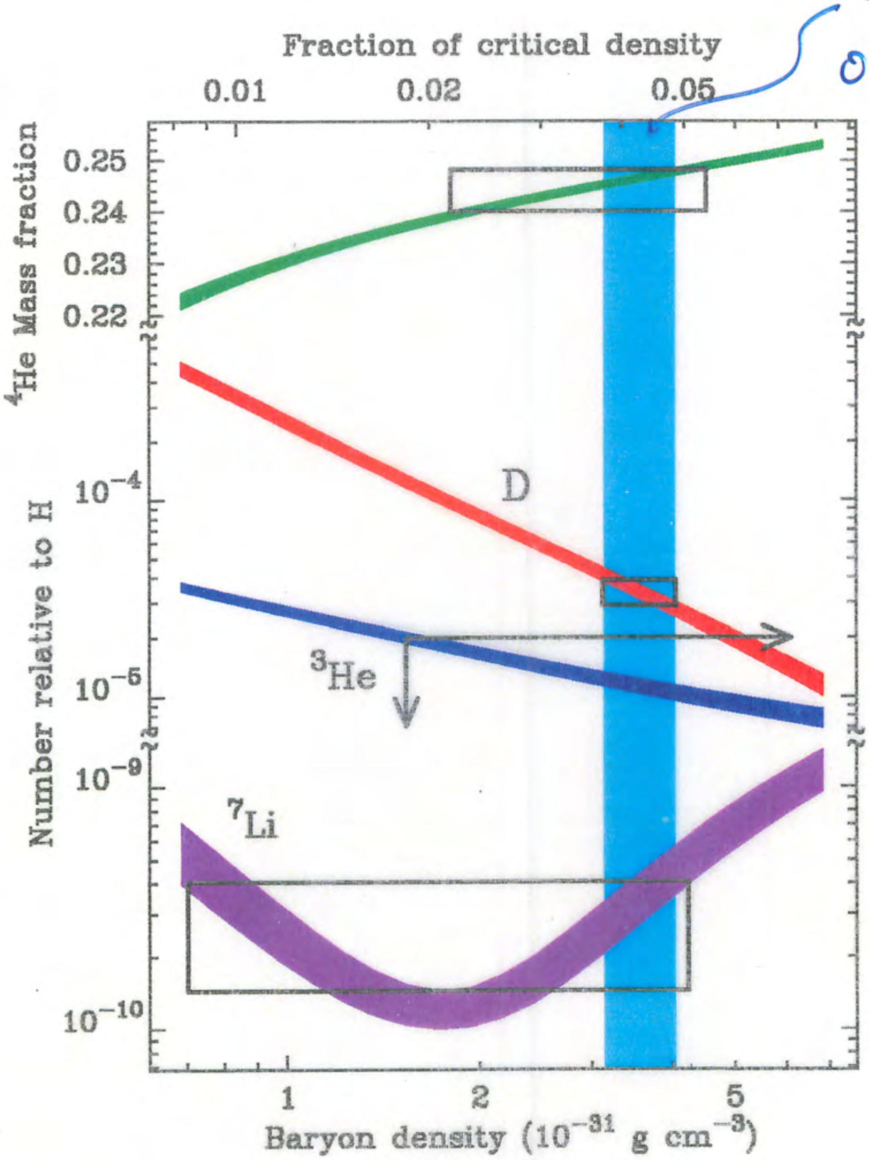
There are no stable nuclei of mass 5 or 8, so elements heavier than ${}^4He$ are difficult to make in the Bang Bang. Trace amounts of ${}^7Li$ and Be are made, but no ${}^{12}C$. This and other elements are produced much later in the interior of stars. Nucleosynthesis is over at about $t \simeq 200\text{s}$



Hans BETHE
(Nobel prize 1967)

BIG BANG NUCLEOSYNTHESIS

allowed by observations



Measurements of deuterium in primordial cloud

$$\Rightarrow \quad \Omega_b h^2 = 0.019 \pm 0.002$$

(Tytler et al 1999)

$$\Omega_b \equiv \frac{\bar{\rho}_{baryon}}{\rho_{crit}}, \quad h \equiv \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}$$

As the graph shows, BBNS only works if $\Omega_b h^2 = 0.020 \pm 0.03$ where:

$$\Omega_b = \frac{\rho_b}{\rho_{\text{crit}}}$$

is the density in baryons and $h = H_o/100\text{km s}^{-1}\ \text{Mpc}^{-1}$. This implies that *baryons cannot account for the measured value of* $\Omega_m = 0.25 \pm 0.02$. Thus the dark matter must be non-baryonic particles.

3. The cosmic microwave background radiation

3.1 The plasma era

From the epoch of BBNS to $t \simeq 358,000$ yrs, the content of the Universe was a plasma, i.e. an ionized gas of mostly H, He, electrons and photons. Hydrogen atoms could not form because the minimum ionization energy of an H atom is 13.6eV and during this time, $KT > 13.6$ eV.

Photons interact strongly with free electrons via Thomson scattering. Their mean free path - $l = 1/n_e\sigma_e$, where n_e is the electron number density and σ_e is the cross section - is small. This keeps the radiation and the electrons (and through this the ions) in thermal equilibrium. Radiation in thermal equilibrium acquires a *black-body spectrum.*

3.2 Decoupling

As the Universe expanded, it cooled and, as a result, the photons lost energy and became less and less able to keep the atoms ionized. When $KT \leq 13.6$ eV, most of the electrons were able to fall into the ground state of H atoms. Once H atoms were present, the photons were no longer able to interact at all. This process is knows as *decoupling* or *(re)-combination.*

The Universe went from being opaque to being completely transparent in a short period of time. Since then the photons have been propagating freely. Decoupling happened where $T \simeq 3000$ K.

3.3 Properties of the CMB

In the plasma epoch, the radiation acquired a black body spectrum. As we shall see shortly, this spectral shape is preserved in time. First, let us recall a few important formulae about black bodies (hereafter bb).

For a bb, the energy density (energy per unit volume) in frequency interval $d\nu$ about ν is

$$\epsilon(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/kT} - 1} \tag{19}$$

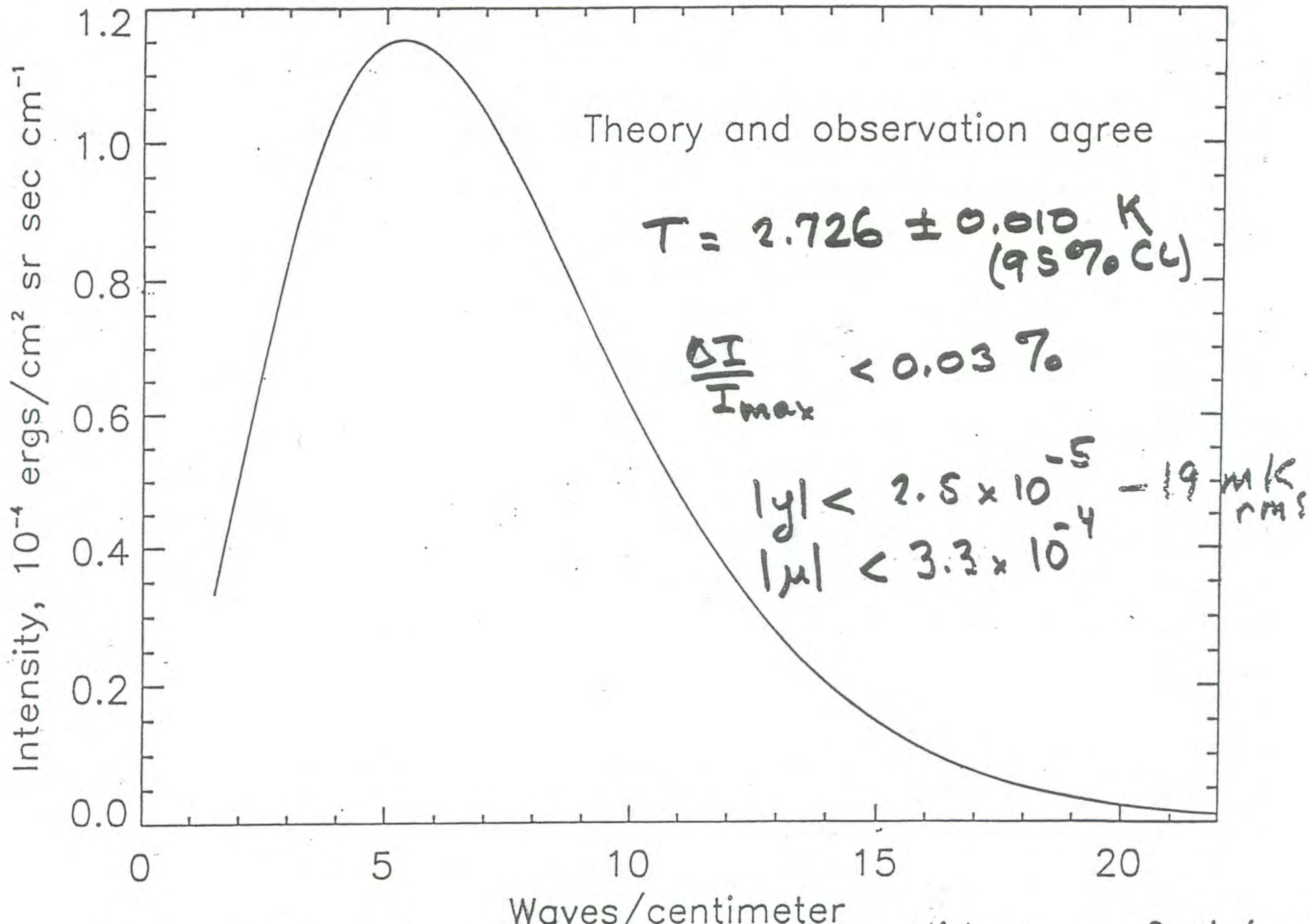
It can be shown that this spectrum gives Wien's law:

$$\lambda_{max}T \propto \frac{T}{\nu_{max}} = \text{const} \tag{20}$$

The total energy is obtained by integrating eqn. (19) over all frequencies. Setting $y = h\nu/kT$ leads to

$$\epsilon_{rad} = \frac{8\pi k^4}{h^3c^3}T^4 \int_0^\infty \frac{y^3 dy}{e^y - 1} \tag{21}$$

This integral (which is not that easy to do) gives $\pi^4/15$. Thus, Cosmic Microwave Background Spectrum from COBE



J.C. Mather, Presented at International School of Astrophysics D. Chalonge "Current Topics in Astrofundamental Physics", Sep. 1994

$$\epsilon_{rad} = \alpha T^4 \equiv \frac{4\sigma}{c^3} T^4 \tag{22}$$

where α is called the radiation constant:

$$\alpha = \frac{\pi^2 k^4}{15\hbar^3 c^3} = 7.565 \times 10^{-16} \mathrm{M}^{-3} \mathrm{K}^{-4} \tag{23}$$

where $\hbar = h/2\pi$ and σ is the Stefan-Boltzman constant.

3.4 Evolution of the CMB

Let us first calculate how the temperature of the radiation changes with z. The number of photons is conserved. Thus, the number density is

$$n_\gamma \propto a^{-3} \propto (1 + z)^3 \tag{24}$$

Each photon has energy

$$E = h\nu \propto h\lambda^{-1} \propto (1 + z) \tag{25}$$

Thus, the energy density of the radiation:

$$\varepsilon_{\mathrm{rad}} = E\, n_\gamma \propto (1 + z)^4 \tag{26}$$

Comparing with eqn. (22) $\Rightarrow T \propto (1 + z)$

So, as $z \downarrow$, the radiation cools. Today,

$$T_0 = 2.728\,\mathrm{K} \tag{27}$$

From eqn. (22), this corresponds to

$$\varepsilon_{\mathrm{rad}}\,(t_0) = 4.9 \times 10^{-14}\,\mathrm{Jm}^{-3}$$

We can compare this to the energy density in baryons. Big Bang nucleosynthesis gives

$$\Omega_b \simeq 0.02\, h^{-2}$$

Using the definition of the critical density, we find

$$\varepsilon_b\,(t_0) = \rho_b\, c^2 \simeq 3.38 \times 10^{-11} \mathrm{Jm}^{-3}$$

Thus, the baryons have $\sim 10^3$ more energy density than the radiation. The typical energy of a CMB photon is

$$\bar{E}_{\mathrm{rad}} \simeq 3k\,T = 7.0 \times 10^{-4}\,\mathrm{eV} \tag{28}$$

while the (rest mass-) energy of a baryon is E_b = 939 MeV.

Thus, we find

$$\frac{n_\gamma}{n_b} = \frac{E_b}{\bar{E}_{\rm rad}} \frac{\varepsilon_{\rm rad}}{\varepsilon_b} = 1.7 \times 10^9 \tag{29}$$

which is the number predicted by baryosynthesis.

Note that since $\nu \propto (1 + z)$ and $T \propto (1 + z)$, then from eqn. (19) the black body spectral shape is preserved as the radiation propagates. That is the fact that CMB is is a black body today, tells us it was a black body at decoupling (but at a higher temperature). Thus the Universe was in thermal equilibrium at early times.

DIAGRAM

Finally, note that although today $\varepsilon_b \simeq 10^3 \varepsilon_{\rm rad}$, i.e. the universe is matter-dominated, this was not always the case because

$$\frac{\varepsilon_{\rm rad}}{\varepsilon_b} \propto \frac{(1 + z)^4}{(1 + z)^3} \propto (1 + z) \tag{30}$$

So, at early times $\varepsilon_{\rm rad} \gg \varepsilon_b$ implying that the Universe was radiation-dominated (ie a fireball).

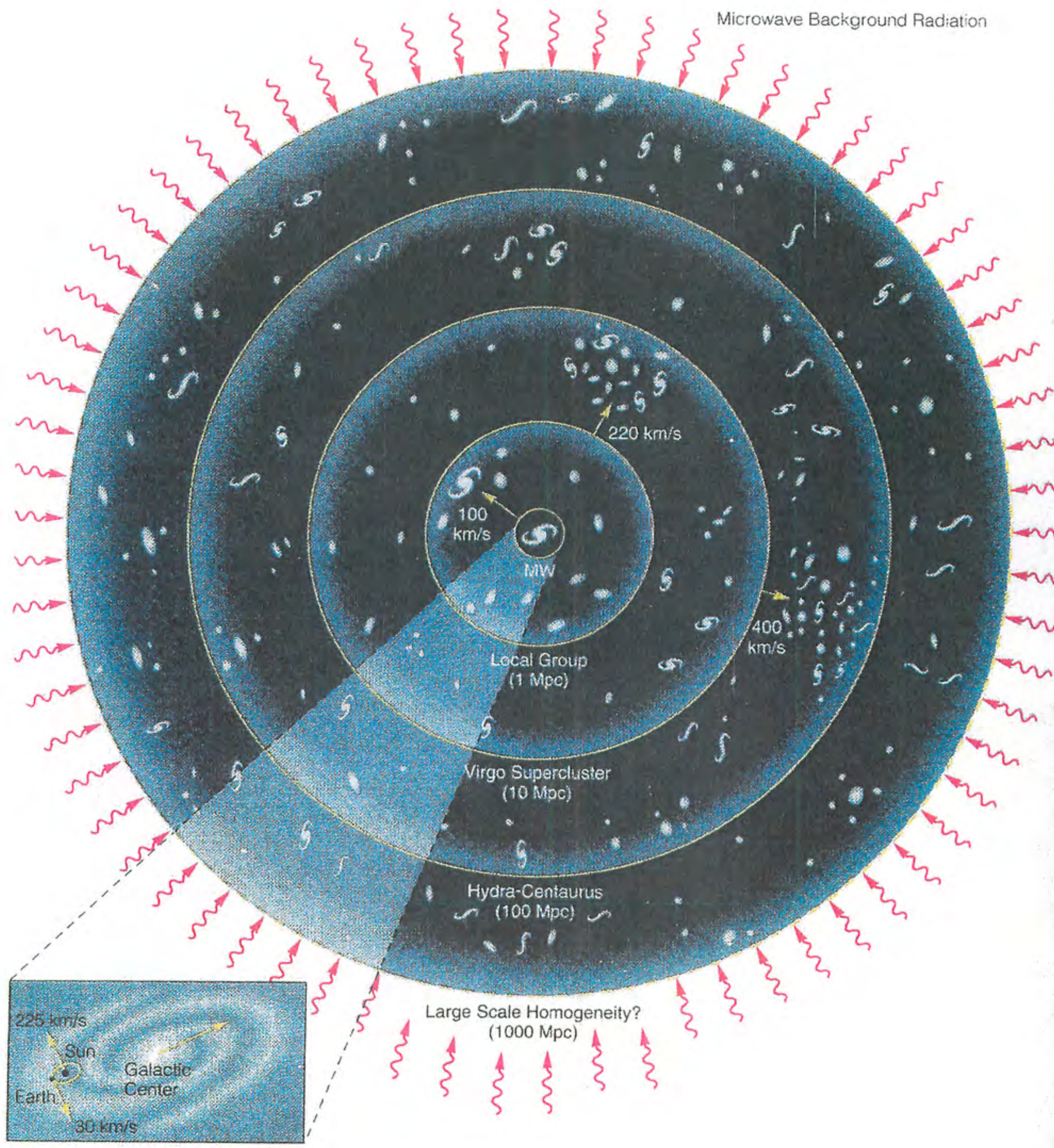
4. The origin of cosmic structure

4.1 Inflation

At early times the Universe is thought to have gone through a phase of exponential expansion called inflation. This was triggered by the likely presence of quantum scalar fields that were initially trapped in the "false vacuum" and eventually decayed to the true vaccum. Inflation makes 2 fundamental predictions:

(1) The universe has a flat geometry: $\Omega_m + \Omega_\Lambda = 1$

(2) During inflation quantum fluctuations in the energy density are stretched to macroscopic scales. These are "scale-invariant" and have a Gaussian distribution of amplitudes. These fluctuations are the progenitors of galaxies and other cosmic structures.


Microwave Background Radiation
220 km/s
100 km/s
MW
400 km/s
Local Group (1 Mpc)
Virgo Supercluster (10 Mpc)
Hydra-Centaurus (100 Mpc)
Large Scale Homogeneity? (1000 Mpc)
225 km/s
Sun
Galactic Center
Earth
30 km/s

4.2 Temperature fluctutations in the CMB

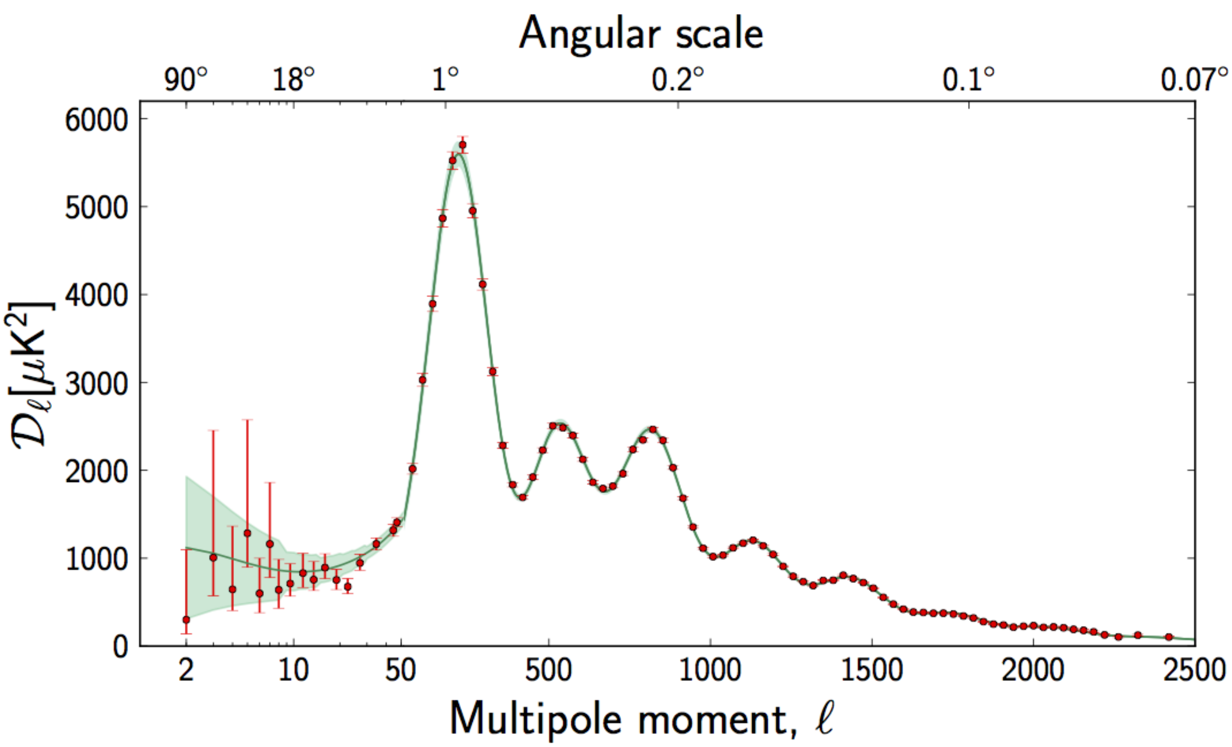
The quantum fluctuations involve all the material content of the Universe: dark matter, baryons and radiation. (The baryons are locked in to the radiation through Thomson scattering.) The temperature fluctuations give rise to a pattern of cold & hot spots in maps of the CMB temperature. There are 3 main effects:

(a) The Sachs-Wolfe effect:
γs emitted from the bottom of potential wells have to climb out of them and as a result they loose energy $\rightarrow$ cold spots

(b) Adiabatic perturbations:
γs are compressed in regions where gas pressure is high $\rightarrow$ hot spots

(c) Doppler effect:
The density pertubations cause the bayrons to flow. γs scattering off the electrons suffer a Doppler effect which results in a temperature fluctuation.

Acoustic peaks are created when the coherently oscillating photon-baryon fluid is frozen at recombination and the sound speed drops. The amplitude of CMB fluctuations is $\Delta T/T \sim 10^{-5}$



4.3 The evolution of fluctuations

The small fluctuations generated during the early universe subsequently grow due as a result of the process of *gravitational instability*. The gravitational effect of the dark matter amplifies initially small fluctuations. To see how this works, think of an overdense region as a patch of a closed universe:

DIAGRAM

Dark matter halos collapse and baryons fall into the potential wells created by the halos. They subsequently cool and fragment to form stars.

DIAGRAM

In the cold dark matter model, there are fluctuations on all scales and these are larger on smaller scales. As a result the first structures that form are subgalactic fragments. These collide and merge building up larger and larger structures in a process known as *hierarchical clustering*.