

### 2.4 The cosmic microwave background radiation

#### 2.4.1 The plasma era

#### 2.4.2 Properties of the CMB

In the plasma epoch, the radiation acquired a black body spectrum. As we shall see shortly, this spectral shape is preserved in time. First, let us recall a few important formulae about black bodies (hereafter bb).

For a bb, the energy density (energy per unit volume) in frequency interval $d\nu$ about $\nu$ is

$$\epsilon(\nu)d\nu = \frac{8\pi \hbar}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$  \hspace{1cm} (2.4)

It can be shown that this spectrum gives Wien’s law:

$$\lambda_{\text{max}} T \propto \frac{T}{\nu_{\text{max}}} = \text{const}$$  \hspace{1cm} (2.5)

The total energy is obtained by integrating (2.4) over all frequencies. Setting $y = h\nu/kT$ leads to

$$\epsilon_{\text{rad}} = \frac{8\pi k^4}{h^3 c^3} T^4 \int_0^\infty \frac{y^3 dy}{e^y - 1}$$  \hspace{1cm} (2.6)

This integral (which is not that easy to do) gives $\pi^4/15$. Thus

$$\epsilon_{\text{rad}} = \alpha T^4 = \frac{4\sigma}{c^3} T^4$$  \hspace{1cm} (2.7)

where $\alpha$ is called the radiation constant:

$$\alpha = \frac{\pi^2 k^4}{15 h^3 c^3} = 7.565 \times 10^{-16} \text{M}^{-3} \text{K}^{-4}$$  \hspace{1cm} (2.8)

where $h = h/2\pi$ and $\sigma$ is the Stefan-Boltzmann constant.
2.4.3 Evolution of the CMB

Let us first calculate how the temperature of the radiation changes with $z$. The number of photons is conserved. Thus, using (1.6), the number density is

$$n_\gamma \propto a^{-3} \propto (1 + z)^3$$  \hspace{1cm} (2.9)

Each photon has energy

$$E = h\nu \propto h\lambda^{-1} \propto (1 + z)$$  \hspace{1cm} (2.10)

Thus, the energy density of the radiation:

$$\varepsilon_{\text{rad}} = E n_\gamma \propto (1 + z)^4$$  \hspace{1cm} (2.11)

Comparing with (2.7) \Rightarrow $T \propto (1 + z)$

So, as $z \downarrow$, the radiation cools. Today,

$$T_0 = 2.728 \text{ K}$$  \hspace{1cm} (2.12)

From (2.7), this corresponds to

$$\varepsilon_{\text{rad}}(t_0) = 4.9 \times 10^{-14} \text{ J m}^{-3}$$

We can compare this to the energy density in baryons. Big Bang nucleosynthesis gives

$$\Omega_b \simeq 0.02 \hbar^{-2}$$

Using the definition of the critical density, we find

$$\varepsilon_b(t_0) = \rho_b c^2 \simeq 3.38 \times 10^{-11} \text{ J m}^{-3}$$

Thus, the baryons have $\sim 10^3$ more energy density than the radiation. The typical energy of a CMB photon is

$$\bar{E}_{\text{rad}} \simeq 3kT = 7.0 \times 10^{-4} \text{ eV}$$  \hspace{1cm} (2.13)

while the (rest mass-) energy of a baryon is $E_b = 939$ MeV.
Thus, we find
\[
\frac{n_\gamma}{n_b} = \frac{E_b}{E_{\text{rad}}} \frac{\varepsilon_{\text{rad}}}{\varepsilon_b} = 1.7 \times 10^9 \tag{2.14}
\]
which is the number predicted by baryosynthesis.

Note that since \( \nu \propto (1 + z) \) and \( T \propto (1 + z) \), then from (2.4) the black body spectral shape is preserved as the radiation propagates. That is the fact that CMB is a black body today, tells us it was a black body at decoupling (but at a higher temperature). Thus the Universe was in thermal equilibrium at early times.

\[\text{DIAGRAM}\]

Finally, note that although today \( \varepsilon_b \simeq 10^3 \varepsilon_{\text{rad}} \), i.e. the universe is matter-dominated, this was not always the case because
\[
\frac{\varepsilon_{\text{rad}}}{\varepsilon_b} \propto \frac{(1 + z)^4}{(1 + z)^3} \propto (1 + z) \tag{2.15}
\]
So, at early times \( \varepsilon_{\text{rad}} \gg \varepsilon_b \) implying that the Universe was radiation-dominated (i.e. a fireball).
2.5 The origin of cosmic structure

2.5.1 Inflation

At early times the Universe is thought to have gone through a phase of exponential expansion called inflation. This was triggered by the likely presence of quantum scalar fields that were initially trapped in the “false vacuum” and eventually decayed to the true vacuum. Inflation makes 2 fundamental predictions:

1. The universe has a flat geometry: $\Omega_m + \Omega_\Lambda = 1$

2. During inflation quantum fluctuations in the energy density are stretched to macroscopic scales. These are “scale-invariant” and have a Gaussian distribution of amplitudes. These fluctuations are the progenitors of galaxies and other cosmic structures.

2.5.2 Temperature fluctuations in the CMB

The quantum fluctuations involve all the material content of the Universe: dark matter, baryons and radiation. (The baryons are locked in to the radiation through Thomson scattering.) The temperature fluctuations give rise to a pattern of cold & hot spots in maps of the CMB temperature. There are 3 main effects:

(a) The Sachs-Wolfe effect:
\[ \gamma \text{ emitted from the bottom of potential wells have to climb out of them and as a result they lose energy} \rightarrow \text{ cold spots} \]

(b) Adiabatic perturbations:
\[ \gamma \text{ are compressed in regions where gas pressure is high} \rightarrow \text{ hot spots} \]

(c) Doppler effect:
The density perturbations cause the baryons to flow. $\gamma$s scattering off the electrons suffer a Doppler effect which results in a temperature fluctuation.

Acoustic peaks are created when the coherently oscillating photon-baryon fluid is frozen at recombination and the sound speed drops. The amplitude of CMB fluctuations is $\Delta T/T \sim 10^{-5}$
2.5.3 The evolution of fluctuation

The small fluctuations generated during the early universe subsequently grow due as a result of the process of gravitational instability. The gravitational effect of the dark matter amplifies initially small fluctuations. To see how this works, think of an overdense region as a patch of a closed universe:

DIAGRAM

Dark matter halos collapse and baryons fall into the potential wells created by the halos.
They subsequently cool and fragment to form stars.

In the cold dark matter model, there are fluctuations on all scales and these are larger on smaller scales. As a result the first structures that form are subgalactic fragments. These collide and merge building up larger and larger structures in a process known as *hierarchical clustering*.